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A Repetitive Group Sampling Plan for Truncated Life Tests Using Transmuted Exponentiated Exponential Distribution and Transmuted Generalized Rayleigh Distribution

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Abstract - In this article, a repetitive group acceptance sampling plan is developed for a truncated life test when the lifetime of an item follows transmuted exponentiated exponential distribution and transmuted generalized rayleigh distribution. The minimum sample sizes are determined when the consumer's risk and the test termination time are specified. The operating characteristic values according to various quality levels are calculated. The results are explained with examples. A comparative study of repetitive group sampling plans for transmuted exponentiated exponential distribution and transmuted generalized rayleigh distribution is also obtained.

Keywords-Transmuted Exponentiated Exponential Distribution, Transmuted Generalized Rayleigh Distribution, Repetitive Group Sampling Plan, Consumer's risk, Operating Characteristics (OC), Producer's risk, Truncated Life Test.

1. INTRODUCTION

Quality Control can be defined as the process through which we measure actual quality performance, compare it with the quality standard set and act on the difference in order to improve the product's quality. The reputation of companies depends upon the high reliability of their products. These companies complete with each other on the basis of quality and reliability. Thus quality control has become one of the most important tools to differentiate between competitive enterprises in a global business market. In order to control the quality of the purchased goods, two major alternatives are open to a buyer. One is the complete inspection, in which every single item in the lot is inspected and tested. This is often impractical, uneconomical or impossible. Secondly, the practical inspection in which a sample of item is taken which is inspected and tested and the whole lot is accepted or rejected depending on whether few or many defective items are found in the sample. Quality control is the use of techniques and activities to achieve, sustain and improve the quality of a product or service.

Acceptance sampling procedures play an important role in improving the quality. The basic aim of all companies in this world is to improve the quality of their products. The high quality product has the high probability of acceptance. In a time-truncated sampling plan, a random sample is selected from a lot of products and put on the test where the number of failures is recorded until the pre-specified time. If the number of failures observed is not greater than the specified acceptance number, then the lot will be accepted. Two risks are always attached to an acceptance sampling. The probability of rejecting the good lot is known as the type-I error (producer's risk). The probability of accepting the bad lot is known as the type-II error (consumer's risk). An acceptance sampling plan should be designed so that both risks are smaller than the required values. An acceptance sampling plan involves quality contracting on product orders between the producer's risk and consumer's risk. Sherman in 1965 was the one who introduced the repetitive group sampling plan. According to him the attribute repetitive group plan is more efficient than the single sampling plan even its operation is similar to sequential sampling. Repetitive group sampling is not very common in life tests. This paper deals with a new method of sampling using repetitive group sampling plan and lifetime distributions which is very effective in reducing the sample size.

In this paper we propose a plan to find the probability of acceptance for the repetitive group acceptance sampling assuming the experiment is truncated at pre-assigned time and lifetime follows Transmuted Exponentiated Exponential Distribution and Transmuted Generalized Rayleigh Distribution. The probability density function (pdf) and the cumulative distribution function (cdf) of the Transmuted Exponentiated Exponential Distribution are given by

$$f(t;\ \alpha,\sigma,\lambda) = \alpha\sigma(1-e^{-t\sigma})^{\alpha-1}e^{-t\sigma}\ [1+\lambda-2\lambda(1-e^{-t\sigma})^{\alpha}] \text{ --------------(1)}$$

$$F(t;\ \alpha,\sigma,\lambda) = (1-e^{-t\sigma})^{\alpha}[1+\lambda-\lambda(1-e^{-t\sigma})^{\alpha}] \text{ -------------------(2)}$$

where, $0<t<\infty$, $\alpha>0$, $\sigma>0$, $|\lambda|\leq 1$, here σ is the scale parameter and α,λ are the shape parameters which are fixed as ($\alpha=2$, $\lambda=1$).

The probability density function (pdf) and the cumulative distribution function (cdf) of the Transmuted Generalized Rayleigh Distribution are given by

$$f(t;\ \alpha,\sigma,\lambda) = 2\alpha\sigma^2 te^{-(\sigma t)^2}\left(1-e^{-(\sigma t)^2}\right)^{\alpha-1}\left[1+\lambda-2\lambda\left(1-e^{-(\sigma t)^2}\right)^{\alpha}\right] \text{ ------------(3)}$$

$$F(t;\ \alpha,\sigma,\lambda) = \left(1-e^{-(\sigma t)^2}\right)^{\alpha}\left[1+\lambda-\lambda\left(1-e^{-(\sigma t)^2}\right)^{\alpha}\right] \text{ --------------(4)}$$

where, $0<t<\infty$, $\alpha>0$, $\sigma>0$, $|\lambda|\leq 1$, here σ is the scale parameter and α,λ are the shape parameters which are fixed as ($\alpha=2$, $\lambda=1$).

If some other parameters are involved, then they are assumed to be known, for an example, if shape parameter of a distribution is unknown it is very difficult to design the acceptance sampling plan. In quality control analysis, the scale parameter is often called the quality parameter or characteristics parameter. Therefore it is assumed that the distribution function depends on time only through the ratio of t/ σ.

2. DESIGN OF THE PROPOSED PLAN

Sherman (1965) has introduced repetitive group sampling plan in which a sample is drawn and the number of defective is counted. According to a fixed criteria, the lot is either accepted, rejected or the sample is completely disregarded and one has to begin with a new sample in order to sentence a lot. This is continued until the fixed criterion indicates us to either accept or reject the lot.

The operating procedure for a repetitive group sampling plan is stated as follows:

1. Select a random sample of size 'n' from a lot of size 'N'.
2. Inspect all the articles included in the sample. Let 'd' be the number of defectives in the sample.
3. If $d \leq c_1$, accept the lot
   If $d > c_2$, reject the lot
   If $c_1 < d \leq c_2$, repeat the steps 1, 2 and 3.

The following is the operation characteristic function for the repetitive group sampling plan

$$L(P) = \frac{P_a}{P_a+P_r} \text{ ---------------(5)}$$

Here P_a and P_r is the binomial model and equation (5) becomes

$$L(P) = \frac{\sum_{i=0}^{c_1}\binom{n}{i}P^i(1-P)^{n-i}}{1-\sum_{i=0}^{c_2}\binom{n}{i}P^i(1-P)^{n-i}+\sum_{i=0}^{c_1}\binom{n}{i}P^i(1-P)^{n-i}} \text{ -------------(6)}$$

Where in equation (5),

$$P_a = \sum_{i=0}^{c_1}\binom{n}{i}P^i(1-P)^{n-i} \quad \text{and}$$

$$P_r = 1-\sum_{i=0}^{c_2}\binom{n}{i}P^i(1-P)^{n-i}$$

Where P is the failure probability. By fixing the time termination ratios t/σ_0 as 0.628, 0.912, 1.257, 1.571, 2.356, 3.141, 3.927 and 4.712, the consumer's risk β as 0.25, 0.10, 0.05 and 0.01 and the mean ratios σ/σ_0 as 2, 4, 6, 8, 10 and 12, one can find the size of the sample size n by substituting the failure probability p in the equation (6) and using the following inequality.

$$L(P) \leq \beta \text{ ------------(7)}$$

The sample size generated using repetitive group sampling plan for the transmuted exponentiated exponential distribution and transmuted generalized rayleigh distribution are presented in table 1 and 2 respectively, and their corresponding operating characteristic values are presented in table 3 and 4 respectively.

Table 1: Minimum sample size for repetitive group sampling plan when the lifetime of the items follows the transmuted exponentiated exponential distribution

β	c_1	c_2	t/σ_0							
			0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.25	0	1	6	3	3	2	2	2	2	2
	0	2	8	4	3	3	3	3	3	3
	0	3	9	5	4	4	4	4	4	4
	0	4	11	6	5	5	5	5	5	5
0.10	0	1	8	5	3	2	2	2	2	2
	0	2	9	5	4	3	3	3	3	3
	0	3	11	6	5	4	4	4	4	4
	0	4	12	7	5	5	5	5	5	5
0.05	0	1	10	6	4	4	2	2	2	2
	0	2	11	6	4	4	3	3	3	3
	0	3	12	7	5	4	4	4	4	4
	0	4	13	8	6	5	5	5	5	5
0.01	0	1	15	8	5	4	2	2	2	2
	0	2	15	8	5	4	3	3	3	3
	0	3	16	9	6	4	4	4	4	4
	0	4	16	9	6	5	5	5	5	5

Table 2: Minimum sample size for repetitive group sampling plan when the lifetime of the items follows the transmuted generalized rayleigh distribution

β	c_1	c_2	t/σ_0							
			0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.25	0	1	13	4	2	2	2	2	2	2
	0	2	16	5	3	3	3	3	3	3
	0	3	19	6	4	4	4	4	4	4
	0	4	21	7	5	5	5	5	5	5
0.10	0	1	18	5	2	2	2	2	2	2
	0	2	20	6	3	3	3	3	3	3
	0	3	23	7	4	4	4	4	4	4

	0	4	25	8	5	5	5	5	5	5
0.05	0	1	22	6	3	2	2	2	2	2
	0	2	24	7	3	3	3	3	3	3
	0	3	26	7	4	4	4	4	4	4
	0	4	28	8	5	5	5	5	5	5
0.01	0	1	32	8	4	2	2	2	2	2
	0	2	33	9	4	3	3	3	3	3
	0	3	34	9	5	4	4	4	4	4
	0	4	36	10	5	5	5	5	5	5

Table 3: Probability of acceptance for repetitive group sampling plans with c_1=0 and c_2=2 when the lifetime of the items follows the transmuted exponentiated exponential distribution

β	t/σ	n	σ/σ_0					
			2	4	6	8	10	12
0.25	0.628	8	0.9379	0.9988	0.9999	1.0000	1.0000	1.0000
	0.912	4	0.9645	0.9993	0.9999	1.0000	1.0000	1.0000
	1.257	3	0.9465	0.9989	0.9999	1.0000	1.0000	1.0000
	1.571	3	0.8221	0.9963	0.9996	0.9999	1.0000	1.0000
	2.356	3	0.2482	0.9629	0.9963	0.9993	0.9998	0.9999
	3.141	3	0.0367	0.8224	0.9810	0.9963	0.9990	0.9996
	3.927	3	0.0054	0.5302	0.9329	0.9868	0.9963	0.9987
	4.712	3	0.0008	0.2482	0.8223	0.9629	0.9895	0.9963
0.10	0.628	9	0.9074	0.9983	0.9998	1.0000	1.0000	1.0000
	0.912	5	0.9117	0.9983	0.9998	1.0000	1.0000	1.0000
	1.257	4	0.8014	0.9957	0.9996	0.9999	1.0000	1.0000
	1.571	3	0.8221	0.9963	0.9996	0.9999	1.0000	1.0000
	2.356	3	0.2482	0.9629	0.9963	0.9993	0.9998	0.9999
	3.141	3	0.0367	0.8224	0.9810	0.9963	0.9990	0.9996
	3.927	3	0.0054	0.5302	0.9329	0.9868	0.9963	0.9987
	4.712	3	0.0008	0.2482	0.8223	0.9629	0.9895	0.9963
0.05	0.628	11	0.8253	0.9966	0.9997	0.9999	1.0000	1.0000
	0.912	6	0.8304	0.9965	0.9997	0.9999	1.0000	1.0000
	1.257	4	0.8014	0.9957	0.9996	0.9999	1.0000	1.0000
	1.571	3	0.8221	0.9963	0.9996	0.9999	1.0000	1.0000

	2.356	3	0.2482	0.9629	0.9963	0.9993	0.9998	0.9999
	3.141	3	0.0367	0.8224	0.9810	0.9963	0.9990	0.9996
	3.927	3	0.0054	0.5302	0.9329	0.9868	0.9963	0.9987
	4.712	3	0.0008	0.2482	0.8223	0.9629	0.9895	0.9963
0.01	0.628	15	0.6044	0.9903	0.9991	0.9998	0.9999	1.0000
	0.912	8	0.6102	0.9900	0.9990	0.9998	0.9999	1.0000
	1.257	5	0.5946	0.9891	0.9989	0.9998	0.9999	1.0000
	1.571	4	0.5012	0.9848	0.9985	0.9997	0.9999	1.0000
	2.356	3	0.2482	0.9629	0.9963	0.9993	0.9998	0.9999
	3.141	3	0.0367	0.8224	0.9810	0.9963	0.9990	0.9996
	3.927	3	0.0054	0.5302	0.9329	0.9868	0.9963	0.9987
	4.712	3	0.0008	0.2482	0.8223	0.9629	0.9895	0.9963

Table 4: Probability of acceptance for repetitive group sampling plans with c_1=0 and c_2=2 when the lifetime of the items follows the transmuted generalized rayleigh distribution

β	t/σ	n	σ/σ_0					
			2	4	6	8	10	12
0.25	0.628	16	0.9992	1.0000	1.0000	1.0000	1.0000	1.0000
	0.912	5	0.9990	1.0000	1.0000	1.0000	1.0000	1.0000
	1.257	3	0.9962	1.0000	1.0000	1.0000	1.0000	1.0000
	1.571	3	0.9517	1.0000	1.0000	1.0000	1.0000	1.0000
	2.356	3	0.0913	0.9982	1.0000	1.0000	1.0000	1.0000
	3.141	3	0.0005	0.9519	0.9995	1.0000	1.0000	1.0000
	3.927	3	0.0000	0.5598	0.9939	0.9998	1.0000	1.0000
	4.712	3	0.0000	0.0913	0.9578	0.9982	1.0000	1.0000
0.10	0.628	20	0.9984	1.0000	1.0000	1.0000	1.0000	1.0000
	0.912	6	0.9979	1.0000	1.0000	1.0000	1.0000	1.0000
	1.257	3	0.9962	1.0000	1.0000	1.0000	1.0000	1.0000
	1.571	3	0.9517	1.0000	1.0000	1.0000	1.0000	1.0000
	2.356	3	0.0913	0.9982	1.0000	1.0000	1.0000	1.0000
	3.141	3	0.0005	0.9519	0.9995	1.0000	1.0000	1.0000
	3.927	3	0.0000	0.5598	0.9939	0.9998	1.0000	1.0000
	4.712	3	0.0000	0.0913	0.9578	0.9982	1.0000	1.0000
0.05	0.628	24	0.9971	1.0000	1.0000	1.0000	1.0000	1.0000
	0.912	7	0.9964	1.0000	1.0000	1.0000	1.0000	1.0000
	1.257	3	0.9962	1.0000	1.0000	1.0000	1.0000	1.0000

	1.571	3	0.9517	1.0000	1.0000	1.0000	1.0000	1.0000
	2.356	3	0.0913	0.9982	1.0000	1.0000	1.0000	1.0000
	3.141	3	0.0005	0.9519	0.9995	1.0000	1.0000	1.0000
	3.927	3	0.0000	0.5598	0.9939	0.9998	1.0000	1.0000
	4.712	3	0.0000	0.0913	0.9578	0.9982	1.0000	1.0000
0.01	0.628	33	0.9921	1.0000	1.0000	1.0000	1.0000	1.0000
	0.912	9	0.9911	1.0000	1.0000	1.0000	1.0000	1.0000
	1.257	4	0.9843	1.0000	1.0000	1.0000	1.0000	1.0000
	1.571	3	0.9517	1.0000	1.0000	1.0000	1.0000	1.0000
	2.356	3	0.0913	0.9982	1.0000	1.0000	1.0000	1.0000
	3.141	3	0.0005	0.9519	0.9995	1.0000	1.0000	1.0000
	3.927	3	0.00008	0.5598	0.9939	0.9998	1.0000	1.0000
	4.712	3	0.00003	0.0913	0.9578	0.9982	1.0000	1.0000

3. EXAMPLE

Assume that an experimenter wants to establish that the lifetime of the product produced in the factory ensures that the true unknown mean life is at least 1000 hours. It is desired to stop the experiment at 1571 hours. It is assumed that c_1=0, c_2=2 and β=0.25. Based on consumer's risk values and the time termination ratio, the minimum sample size is determined using the repetitive group acceptance sampling plan for truncated life test. Following are the results obtained when the lifetime of the test items follows the transmuted exponentiated exponential distribution and transmuted generalized rayleigh distribution, respectively.

Table 5: L(P) values for different lifetime distributions when c_1=0, c_2=2 and β=0.25

σ/σ_0	2	4	6	8	10	12
L(P) values of transmuted exponentiated exponential distribution	0.8221	0.9963	0.9996	0.9999	1.0000	1.0000
L(P) values of transmuted generalized rayleigh distribution	0.9517	1.0000	1.0000	1.0000	1.0000	1.0000

From the above distributions one can see that the transmuted generalized rayleigh distribution is comparatively better than the transmuted exponentiated exponential distribution in case of probability of acceptance when the repetitive group sampling plan is used.

4. CONCLUSION

In this paper a repetitive group sampling plan is developed when the lifetime of the items follow transmuted exponentiated exponential distribution and transmuted generalized rayleigh distribution. Minimum sample size required to accept or reject a submitted lot for a given acceptance number were obtained. It is observed that the sample size decreases as the time termination ratio increases. The operating characteristic values were tabulated. Moreover the operating characteristic values increases when the quality improves. Finally, one can see that the repetitive group sampling plan based on transmuted generalized rayleigh distribution is comparatively better than the repetitive group sampling plan based on transmuted exponentiated exponential distribution. This sampling plan can suggested for the industrial purposes to save time and cost of the life test experiments.

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